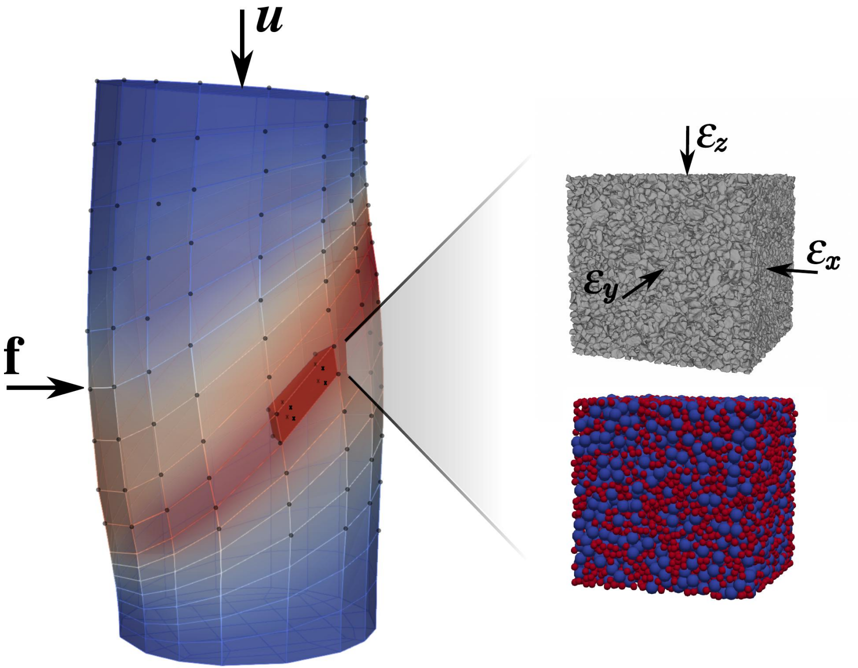
EPFL








# CIVIL-408 Multiscale Modeling in Mechanics

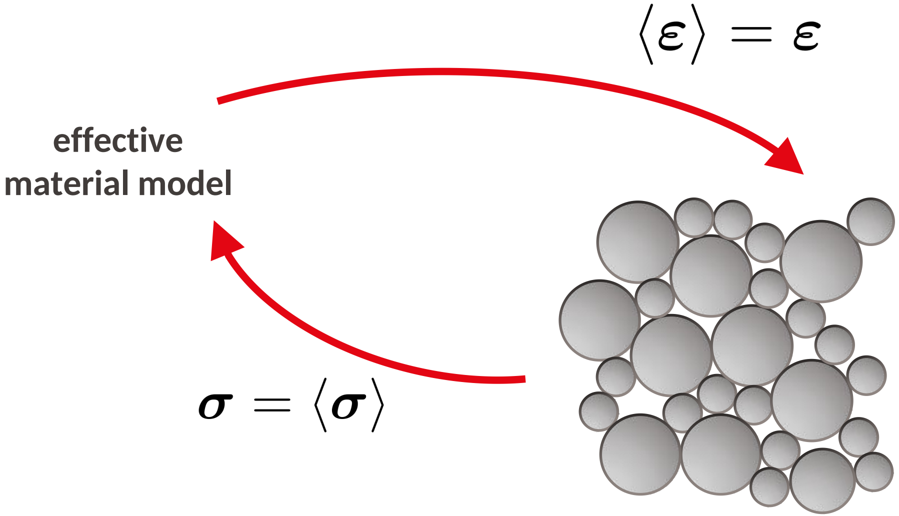
Prof. Kostas Karapiperis

## Exercises - Week 8

# Computational Homogenization

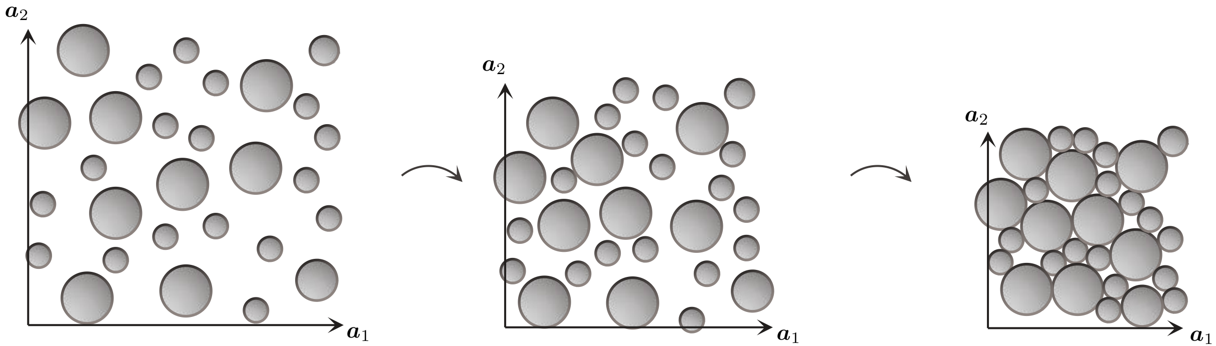
**Strategy**:

- Compute the effective constitutive response of the material by solving a lower-scale boundary value problem from an RVE:

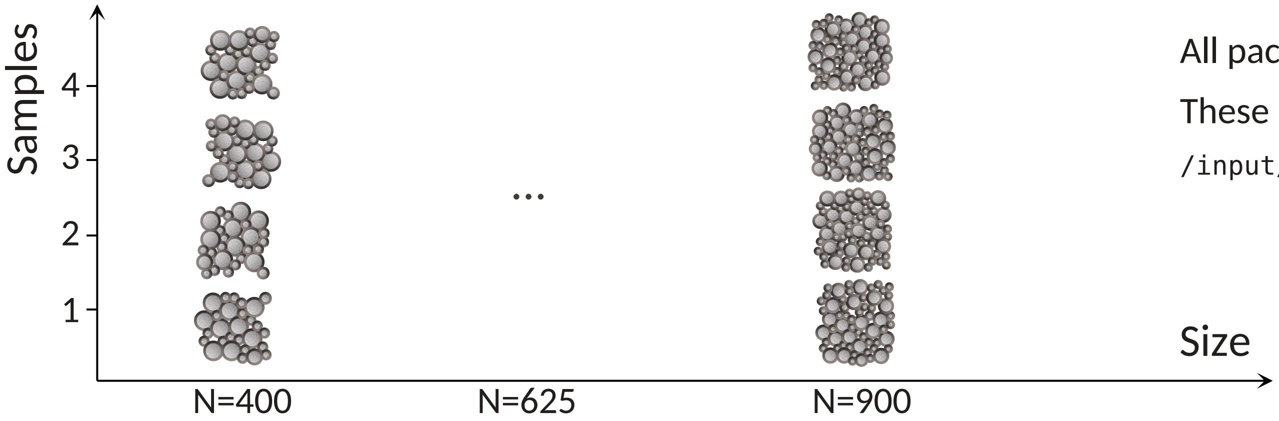


# Recap - Packings

## Particle packing simulation




Using the exercise code, we have compiled many periodic packings for the project:



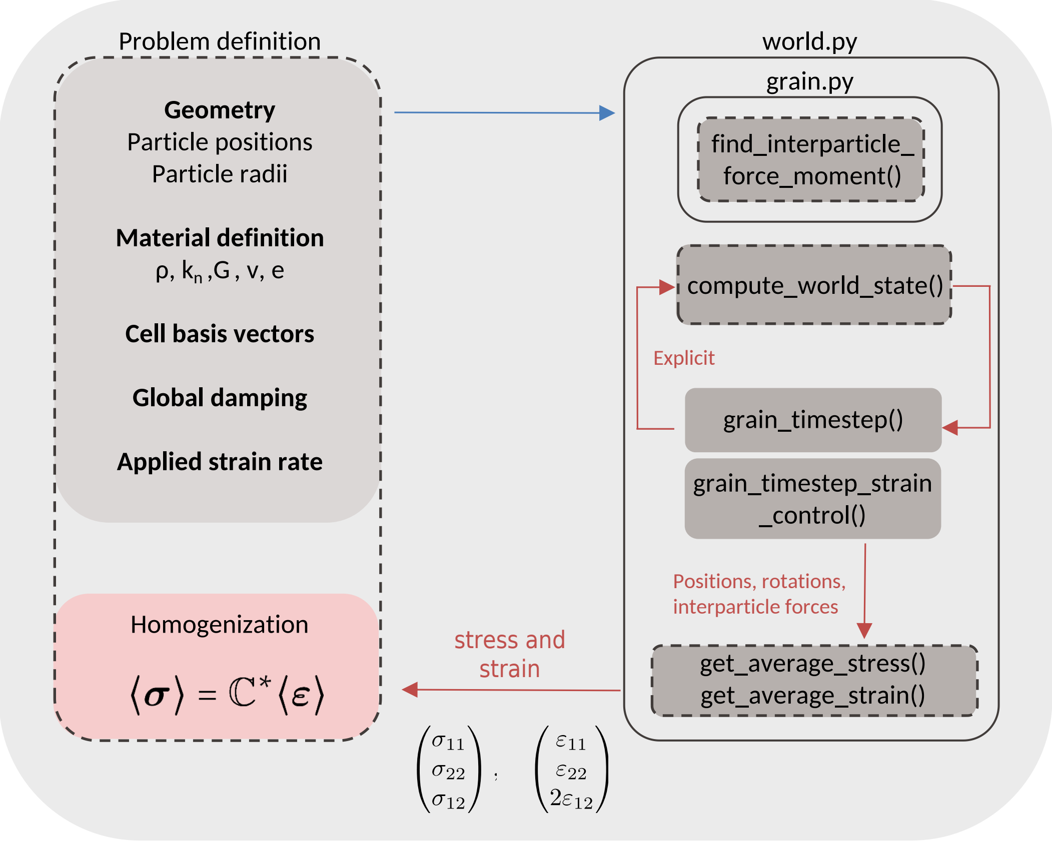

All packings have the same $\phi$

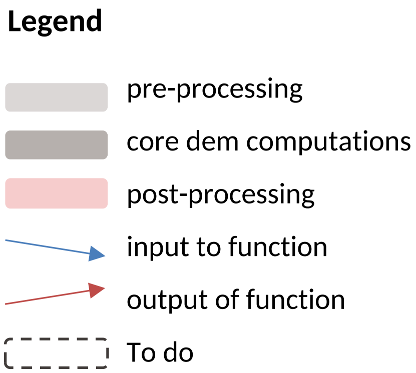
These are found here:

`/input/bidisperse_n=***/sample*`

# Recap - Python implementation

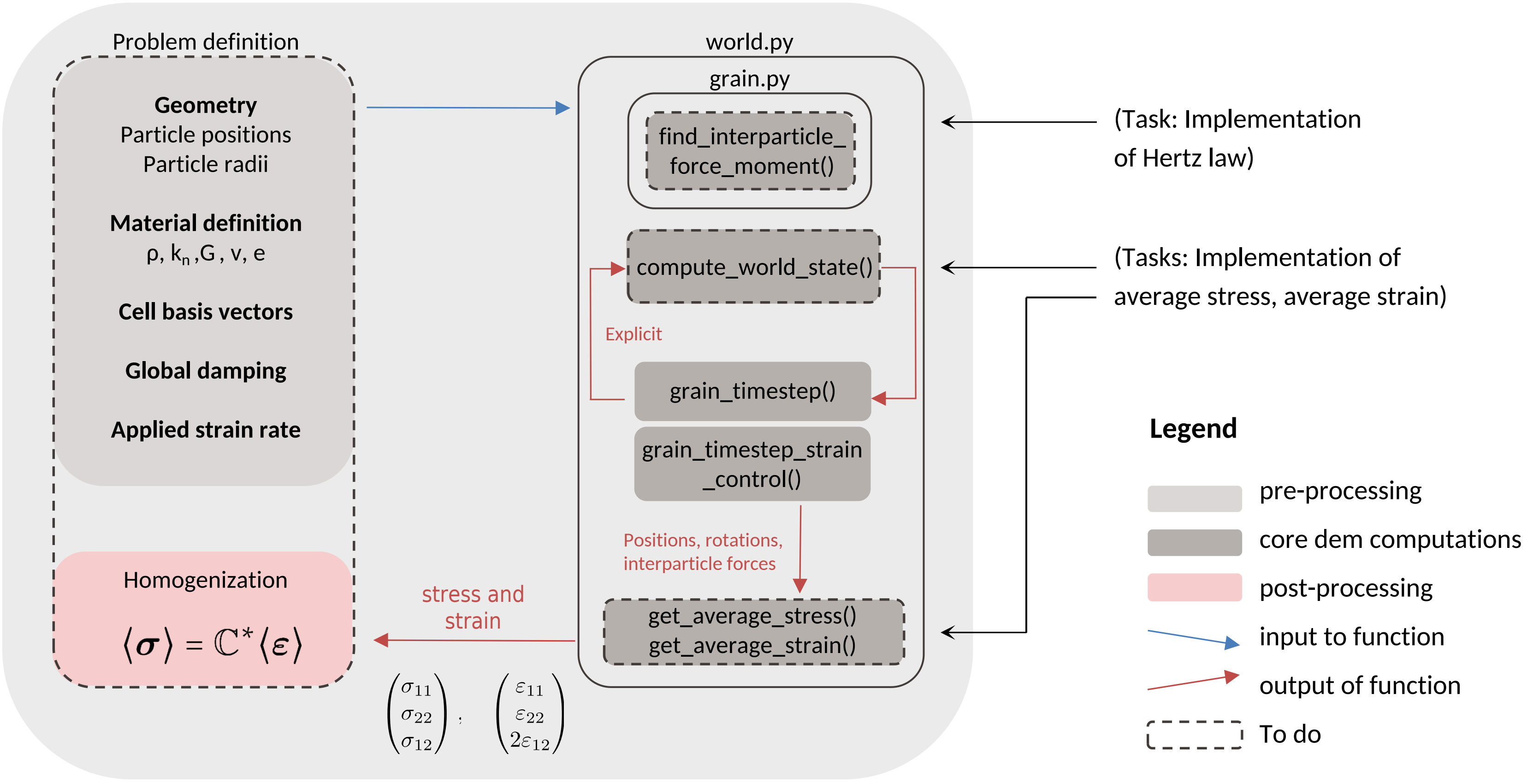
homogenized_stiffness.py/homogenized_bulk_modulus.py



# Recap - Python implementation

homogenized_stiffness.py/homogenized_bulk_modulus.py



# Project 2 - DEM Homogenization



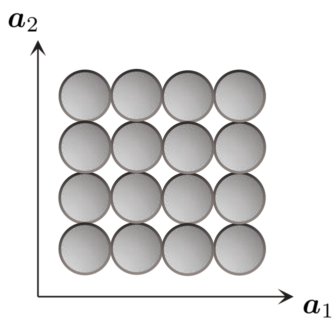
## **A.** Influence of RVE size on a periodic packing (Hookean contact)

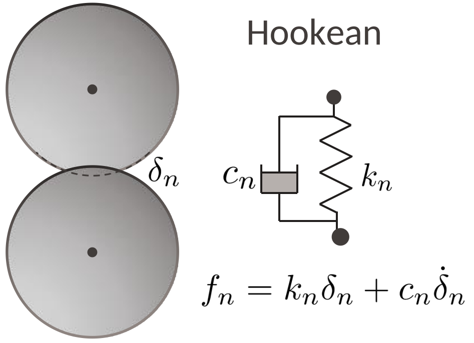
- Impose volumetric compression via periodic boundary conditions $\varepsilon_v$

- Compute the resulting pressure $p$

- Determine the bulk modulus at different levels of compression $K = p/\varepsilon_v$

- Repeat the calculation for two square unit cells (4x4, 8x8)




**Note**: Avoid rate effects by performing slow tests

These monodisperse assemblies are found in:

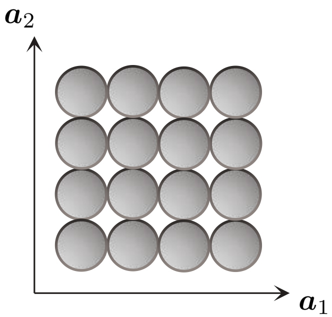
```
/input/monodisperse_n=***/
```

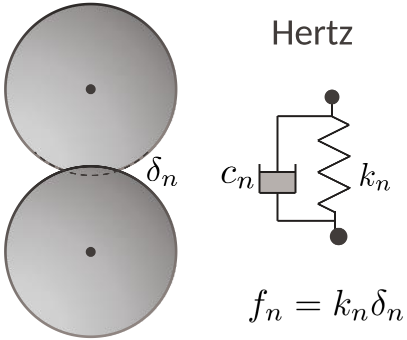
# Project 2 - DEM Homogenization

## **B.** Influence of RVE size on a periodic packing (Hertzian contact)

- Repeat the same procedure as in **A** but for Hertz contact law
- Compare trend with **A**

**Note**: Avoid rate effects by performing slow tests




$$f_n = k_n \delta_n$$

$$k_n = \frac{2G^* \sqrt{2R^*}}{3(1-2\nu^*)} \sqrt{\delta_n}$$

$$R^* = \frac{2R_1 R_2}{R_1 + R_2} \quad G^* = \frac{G_1 + G_2}{2} \quad \nu^* = \frac{\nu_1 + \nu_2}{2}$$

$G = 1 N/m^2$

$v = 0.3$

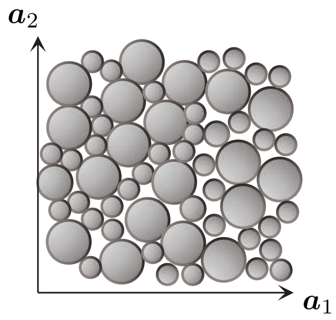
# Project 2 - DEM Homogenization

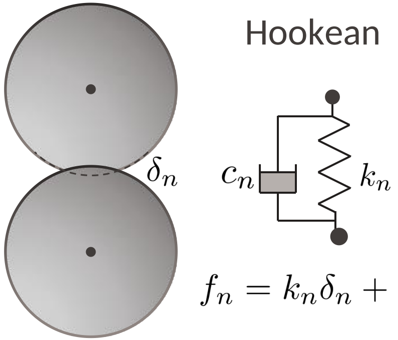
## C. Influence of RVE size on a random bidisperse packing

- Here we compute the entire **homogenized stiffness tensor** following the approach of Project 1.

- We examine **sample convergence**.

- All samples (assemblies) have the **same packing fraction**.




$$f_n = k_n \delta_n + c_n \dot{\delta}_n$$

$k_n$=1N/m

These bidisperse assemblies are found in:

`/input/bidisperse_n=***/sample*`

# Project 2 - DEM Homogenization

K.Karapiperis

## C. Influence of RVE size on a random bidisperse packing

**9 unknown moduli** in 2D:

- apply three (linearly independent strain states)
- compute the three resulting stress states
- solve for the nine unknowns or obtain directly, e.g.,

$$\begin{pmatrix} \langle\sigma_{11}\rangle \\ \langle\sigma_{22}\rangle \\ \langle\sigma_{12}\rangle \end{pmatrix} = \begin{pmatrix} \mathbb{C}^*_{1111} & \mathbb{C}^*_{1122} & \mathbb{C}^*_{1112} \\ \mathbb{C}^*_{2211} & \mathbb{C}^*_{2222} & \mathbb{C}^*_{2212} \\ \mathbb{C}^*_{1211} & \mathbb{C}^*_{1222} & \mathbb{C}^*_{1212} \end{pmatrix} \begin{pmatrix} \langle\varepsilon\rangle \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \mathbb{C}^*_{1111}\langle\varepsilon\rangle \\ \mathbb{C}^*_{2211}\langle\varepsilon\rangle \\ \mathbb{C}^*_{1211}\langle\varepsilon\rangle \end{pmatrix}$$

$$\begin{pmatrix} \langle\sigma_{11}\rangle \\ \langle\sigma_{22}\rangle \\ \langle\sigma_{12}\rangle \end{pmatrix} = \begin{pmatrix} \mathbb{C}^*_{1111} & \mathbb{C}^*_{1122} & \mathbb{C}^*_{1112} \\ \mathbb{C}^*_{2211} & \mathbb{C}^*_{2222} & \mathbb{C}^*_{2212} \\ \mathbb{C}^*_{1211} & \mathbb{C}^*_{1222} & \mathbb{C}^*_{1212} \end{pmatrix} \begin{pmatrix} 0 \\ \langle\varepsilon\rangle \\ 0 \end{pmatrix} = \begin{pmatrix} \mathbb{C}^*_{1122}\langle\varepsilon\rangle \\ \mathbb{C}^*_{2222}\langle\varepsilon\rangle \\ \mathbb{C}^*_{1222}\langle\varepsilon\rangle \end{pmatrix}$$

$$\begin{pmatrix} \langle\sigma_{11}\rangle \\ \langle\sigma_{22}\rangle \\ \langle\sigma_{12}\rangle \end{pmatrix} = \begin{pmatrix} \mathbb{C}^*_{1111} & \mathbb{C}^*_{1122} & \mathbb{C}^*_{1112} \\ \mathbb{C}^*_{2211} & \mathbb{C}^*_{2222} & \mathbb{C}^*_{2212} \\ \mathbb{C}^*_{1211} & \mathbb{C}^*_{1222} & \mathbb{C}^*_{1212} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ \langle\varepsilon\rangle \end{pmatrix} = \begin{pmatrix} \mathbb{C}^*_{1112}\langle\varepsilon\rangle \\ \mathbb{C}^*_{2212}\langle\varepsilon\rangle \\ \mathbb{C}^*_{1212}\langle\varepsilon\rangle \end{pmatrix}$$

# Python notebook





**Let’s move to the Python notebook**